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Coulomb interaction and the Fermi liquid state: solution by bosonization

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Abstract. We investigate the effects of the Coulomb two-body interaction on Fermi liquids via bosonization in two and three spatial dimensions. The Coulomb interaction is singular in the limit of low momentum transfer, and recent interest in the possibility that some singular interactions might destroy the Fermi liquid state motivate us to reexamine it. We calculate the exact boson correlation function to show that the Fermi liquid state is retained in the case of Coulomb interactions in two and three dimensions. Spin and charge degrees of freedom propagate together at the same velocity and collective charge excitations (plasmons) exhibit the expected energy gap in three dimensions. Non-Fermi liquid behaviour occurs in two dimensions, however, for a super-long-range interaction studied recently by Bares and Wen.

The many body problem of fermions interacting via the Coulomb interaction is central to condensed matter physics. The standard method for studying this problem is the summation of ring diagrams which arise in perturbation theory: the random phase approximation (RPA) [1]. Recently, another approach to this problem based on the renormalization group (RG) has been developed [2]. Both the RPA and the RG calculations assume that the Fermi quasiparticle propagator retains the Fermi liquid form with a simple pole. This assumption is shown to be self-consistent since in both approaches the bare Coulomb interaction is screened down to a short-range form. On the other hand, it is desirable to develop tools that do not rely on the intermediate assumption of a Fermi liquid propagator to ascertain whether non-Fermi liquid states are solutions of systems with singular interactions.

Bosonization in dimensions greater than one [3–5] is particularly well suited to the study of singular long-range interactions as the realm in which it is applicable, for low-energy excitations near the Fermi surface, is precisely where these interactions are strongest. Furthermore, bosonization does not rely upon a Fermi liquid form for the quasiparticle propagator; for example, it encompasses the possibility of spin-charge separation. Our results, however, are unambiguous: we find that the Fermi liquid state is *the solution* to the Coulomb problem. By using the bosonization transformation to determine the fermion quasiparticle propagator, we obtain well-known results for the fermion self energy: the imaginary part is proportional to $\omega^2 \ln |\omega|$ in two dimensions and just ω^2 in three dimensions. We emphasize that the bosonization method yields non-perturbative information, so a natural next step would be to use it to study the effects of transverse gauge interactions.

We begin with the bare Hamiltonian for electrons or other fermions interacting via the two-body Coulomb or other long-range longitudinal interaction $V(q)$:

$$H = \sum_k \epsilon_k c_k^{\dagger\alpha} c_{k\alpha} + \frac{1}{2\text{Vol}} \sum_{k,p,q} V(q) c_k^{\dagger\alpha} c_{p+q}^{\dagger\beta} c_{p\beta} c_{k+q\alpha} \quad (1)$$

where $\epsilon_k \equiv k^2/2m$ for the case of a Fermi gas and there is an implicit sum over repeated spin indices α and β . Then we integrate out the high-energy Fermi degrees of freedom with the use of the renormalization group [2, 6]. The resulting low-energy effective theory is expressed in terms of quasiparticles $\psi_{k\alpha}$ which obey canonical anticommutation relations and which are related to the bare electron operators via the wavefunction renormalization factor Z_k :

$$\psi_{k\alpha} = Z_k^{-1/2} c_{k\alpha} \quad (2)$$

for momenta k restricted to a narrow shell of thickness λ around the Fermi surface: $k_F - \lambda/2 < |k| < k_F + \lambda/2$. The resulting low-energy Fermi liquid Hamiltonian is

$$H_{\text{FL}} = \sum_k \tilde{\epsilon}_k \psi_k^\dagger \psi_{k\alpha} + \frac{1}{2\text{Vol}} \sum_{S,T;q} V(q) J(S; -q) J(T; q) + \{\text{regular terms}\}. \quad (3)$$

Here $\tilde{\epsilon}_k \equiv k^2/2m^*$ incorporates the mass renormalization. The charge current in a given Fermi surface patch S (where $S \equiv (\theta, \phi)$ in three dimensions) is defined by

$$J(S; q) \equiv \sum_k \theta(S; k+q) \theta(S; k) \{ \psi_{k+q}^\dagger \psi_{k\alpha} - \delta_{q,0}^3 n_k \}. \quad (4)$$

Here $\theta(S; k) = 1$ if k lies inside a squat box of dimensions $\lambda \times \Lambda^{D-1}$ centred at S and is zero otherwise. The regular terms in (3) do not diverge in the $q \rightarrow 0$ limit and consist of both large- q exchange processes and effective interactions generated by the high-energy electrons that have been integrated out. We assume in the following that no superconducting or charge or spin density wave instabilities arise from the regular terms.

We now review the main aspects of bosonization and refer the reader to two previous papers [4, 5] for more details. In D spatial dimensions, the Fermi quasiparticle fields of spin- σ , ψ_σ , may be expressed [4] in terms of the Abelian boson fields ϕ_σ as

$$\psi_\sigma(S; x) = \frac{1}{\sqrt{\text{Vol}}} \sqrt{\frac{\Omega}{a}} e^{ik_S \cdot x} \exp\left(i \frac{\sqrt{4\pi}}{\Omega} \phi_\sigma(S; x)\right) O(S) \quad (5)$$

where the dependence on time, t , is included implicitly in the spatial coordinates x , and S labels the patch on the Fermi surface with momentum k_S . Vol is the volume of the system, which equals L^D in D dimensions; the factor $\sqrt{\text{Vol}}$ is introduced to keep the fermion anticommutation relations canonical. Both the ψ and ϕ fields live inside the squat box centred on S with height λ in the radial (energy) direction and area Λ^{D-1} along the Fermi surface. These two scales must be small in the following sense: $k_F \gg \Lambda \gg \lambda$. We satisfy these limits by setting $\lambda \equiv k_F/N$ and $\Lambda \equiv k_F/N^\alpha$, where $0 < \alpha < 1$ and $N \rightarrow \infty$. The quantity a in the bosonization formula (5) is a real-space cutoff given by $a \equiv 1/\lambda$. Here $\Omega \equiv \Lambda^{D-1} (L/2\pi)^D$ equals the number of states in the squat box divided by λ . Finally, $O(S)$ is an ordering operator introduced [4, 7] to maintain Fermi statistics in the angular direction along the Fermi surface. (Anticommuting statistics are obeyed automatically in the radial direction.)

With the connection (5) between the fermion and boson fields we may bosonize the free Hamiltonian. The result is quadratic in the ϕ fields:

$$\begin{aligned} H_0 &= v_F \sum_S \int d^D x \psi^\dagger(S; x) \left\{ \frac{\hat{n}_S \cdot \nabla}{i} - k_F \right\} \psi(S; x) \\ &= \frac{2\pi v_F}{\Omega \text{Vol}} \sum_S \int d^D x [(\hat{n}_S \cdot \nabla) \phi(S; x)]^2. \end{aligned} \quad (6)$$

The total bosonized Hamiltonian may be written as $H = H_c + H_s$, exhibiting the factorization into charge and spin sectors. The charge Hamiltonian is bilinear in the current operators [4] $J(\mathbf{S}; \mathbf{q})$:

$$H_c = \frac{1}{2} \sum_{\mathbf{S}, \mathbf{T}} \sum_{\mathbf{q}} V_c(\mathbf{S}, \mathbf{T}; \mathbf{q}) J(\mathbf{S}; -\mathbf{q}) J(\mathbf{T}; \mathbf{q}). \quad (7)$$

Long-range interactions $V(\mathbf{q})$ are incorporated into V_c as matrix elements that couple currents in different patches:

$$V_c(\mathbf{S}, \mathbf{T}; \mathbf{q}) = \frac{1}{2} \Omega^{-1} v_F \delta_{\mathbf{S}, \mathbf{T}}^{D-1} + \frac{1}{\text{Vol}} V(\mathbf{q}). \quad (8)$$

The charge currents are related to the charge boson field $\phi \equiv \phi_\uparrow + \phi_\downarrow$ by $J(\mathbf{S}; \mathbf{x}) = \sqrt{4\pi} \hat{n}_S \cdot \nabla \phi(\mathbf{S}; \mathbf{x})$. They obey the equal-time U(1) current algebra relation [4]:

$$[J(\mathbf{S}; \mathbf{q}), J(\mathbf{T}; \mathbf{p})] = 2\Omega \delta_{\mathbf{S}, \mathbf{T}}^{D-1} \delta_{\mathbf{q}+\mathbf{p}, 0}^D \mathbf{q} \cdot \hat{n}_S. \quad (9)$$

Similarly, the spin sector is described by a Hamiltonian which has a free part

$$H_s = \frac{1}{2} \sum_{\mathbf{S}} \sum_{\mathbf{q}} \frac{v_F}{2\Omega} J_z(\mathbf{S}; -\mathbf{q}) J_z(\mathbf{S}; \mathbf{q}) \quad (10)$$

and an interaction part that depends on the regular Fermi liquid spin-spin coefficients f_s . Here the Abelian spin currents J_z commute with the charge currents and are expressed in terms of the spin boson field $\phi_z \equiv \phi_\uparrow - \phi_\downarrow$ by $J_z(\mathbf{S}; \mathbf{x}) = \sqrt{4\pi} \hat{n}_S \cdot \nabla \phi_z(\mathbf{S}; \mathbf{x})$. The interaction term has two parts: a term that couples the z component of the spin currents in different patches and a term that couples the x and y components which has the form

$$\sum_{\mathbf{S}, \mathbf{T}} f_s(\mathbf{S}, \mathbf{T}) \int d^D x \cos\left(\frac{\sqrt{4\pi}}{\Omega} [\phi_z(\mathbf{S}; \mathbf{x}) - \phi_z(\mathbf{T}; \mathbf{x})]\right). \quad (11)$$

We note that the coefficients f_s are invariant under the renormalization group transformations despite the fact that the above term resembles the form of the bosonized BCS interaction [5]. The crucial difference is that here the boson fields do not appear in pairs at opposite points of the Fermi surface. The β function for the coefficients f_s therefore equals zero. For simplicity we set $f_s(\mathbf{S}, \mathbf{T}) = 0$ in the following.

We now calculate the exact boson Green function in the charge sector. To simplify the calculation we restrict our attention to the case of spherical (circular in two dimensions) Fermi surfaces and set the Fermi velocity equal to one. None of these simplifications is essential. First, we write the charge currents in terms of boson operators that satisfy canonical commutation relations. The choice

$$J(\mathbf{S}; \mathbf{q}) = \sqrt{2\Omega |\hat{n}_S \cdot \mathbf{q}|} [a(\mathbf{S}; \mathbf{q}) \theta(\hat{n}_S \cdot \mathbf{q}) + a^\dagger(\mathbf{S}; -\mathbf{q}) \theta(-\hat{n}_S \cdot \mathbf{q})] \quad (12)$$

with

$$[a(\mathbf{S}; \mathbf{q}), a^\dagger(\mathbf{T}; \mathbf{p})] = \delta_{\mathbf{S}, \mathbf{T}}^{D-1} \delta_{\mathbf{q}, \mathbf{p}}^D \quad (13)$$

and $\theta(x) = 1$ if $x > 0$ and zero otherwise, satisfies the $U(1)$ current commutation relation (9). The momentum–frequency space propagator for the a fields is related to the propagator of the ϕ fields by

$$\langle \phi(S; \mathbf{q}, \omega) \phi(S; -\mathbf{q}, -\omega) \rangle = \frac{\Omega}{2\pi \hat{n}_S \cdot \mathbf{q}} \langle a(S; \mathbf{q}, \omega) a^\dagger(S; \mathbf{q}, \omega) \rangle. \quad (14)$$

As the interaction $V(\mathbf{q})$ has no dependence on the patch indices S and T , a straightforward generalization of our previous calculation [5] for the short-range interaction F_0 allows us to construct the boson propagator. The exact solution of the Dyson equation for the irreducible self-energy is given by

$$\Sigma^I(S; \mathbf{q}, \omega) = \frac{2\Lambda^{D-1} \hat{n}_S \cdot \mathbf{q}}{(2\pi)^D} \frac{V(\mathbf{q})}{1 + V(\mathbf{q})\chi^0(\mathbf{q}, \omega)}. \quad (15)$$

The Lindhard function χ^0 in two dimensions is given by

$$\begin{aligned} \chi^0(x) &= N(0) \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{\cos(\phi)}{\cos(\phi) - x - i\eta \operatorname{sgn}(\omega)} \\ &= N(0) \left(1 - |x| \frac{\theta(x^2 - 1)}{\sqrt{x^2 - 1}} + i|x| \frac{\theta(1 - x^2)}{\sqrt{1 - x^2}} \right) \quad D = 2 \end{aligned} \quad (16)$$

where $x \equiv \omega/|q|$ and $N(0)$ is the density of states at the Fermi energy, which in two dimensions equals k_F/π in units where $v_F = 1$. In three dimensions the Lindhard function is

$$\chi^0(x) = N(0) \left[1 - \frac{1}{2}x \ln\left(\frac{x+1}{x-1}\right) \right] \quad D = 3. \quad (17)$$

Despite the fact that a perturbative expansion [5] has been used as an intermediate step to obtain (15), all terms in the expansion have been summed yielding the exact non-perturbative result valid for arbitrary dimension D . For example, when the interaction is short ranged, in $D = 1$ the resummed expansion yields the well known exact result for a Luttinger liquid [4].

In the $N \rightarrow \infty$ limit in which $q \rightarrow 0$ the self-energy simplifies because $V(q) \rightarrow \infty$. For finite x we obtain

$$\Sigma^I(S; \mathbf{q}, \omega) = \frac{2\Lambda^{D-1} \hat{n}_S \cdot \mathbf{q}}{(2\pi)^D \chi^0(x)}. \quad (18)$$

Screening is apparent at finite x , even in the boson propagator, as the self-energy in the small- q limit no longer depends on $V(q)$. The velocity is renormalized slightly from its bare value of unity, $v_F^r = 1 + O(\Lambda/k_F)^{D-1}$, and the boson lifetime is now finite because of scattering into different patches. Note, however, that these changes represent irrelevant corrections as the self-energy (15) scales to zero as $\Lambda \rightarrow 0$. In particular, the pole in the boson propagator remains unchanged as $N \rightarrow \infty$. In the opposite ω -limit [8] of $x \rightarrow \infty$ the denominator in (15) vanishes at frequencies corresponding to the eigenvalues of the collective mode equation; consequently the self-energy diverges at these frequencies and in the three-dimensional Coulomb case the plasmons acquire a gap as expected [1].

Evidently long-range longitudinal interactions modify the boson propagator. Though BCS scattering processes were ignored, small-angle scattering processes made the boson

lifetime finite. With these results we can use the bosonization formula (5) to infer the fermion quasiparticle self-energy. Since bosonization is carried out in (x, t) space we must carry out three operations. First, we Fourier transform the charge and spin boson propagators into real space and use the Abelian relation $\phi_{\uparrow, \downarrow} = \frac{1}{2}(\phi \pm \phi_c)$ to construct the boson propagator for, say, up spins. Next, the exponential of the resulting expression yields the fermion propagator in real space. Finally, an inverse Fourier transform of the fermion propagator back into momentum space allows us to extract the self-energy.

It is difficult technically to perform these steps in full generality. It will be sufficient for our purposes to estimate the imaginary part of the fermion self-energy. The charge boson Green function may be written in configuration space as

$$iG_c(S; x, t) \equiv \langle \phi(S; x, t)\phi(S; 0, 0) - \phi^2(S; 0, 0) \rangle = i\text{FT}\{G_c(S; k, \omega)\} \\ = i\text{FT}\{G_c^{(0)}(S; k, \omega) + G_c^{(1)}(S; k, \omega)\} \tag{19}$$

where FT represents the Fourier transform operation that converts the variables (k, ω) to (x, t) and also subtracts out the $x = t = 0$ component. The first term, $G_c^{(0)}$, is the free charge boson propagator given in real space by

$$\langle \phi(T; x, t)\phi(T; 0, 0) - \phi^2(T; 0, 0) \rangle = \frac{\Omega^2}{2\pi} \ln\left(\frac{ia}{x \cdot \hat{n}_T - t + i\delta \text{sgn}(t)}\right) \quad |x_{\perp}\Lambda| \ll 1 \\ \rightarrow -\infty \quad |x_{\perp}\Lambda| \gg 1 \tag{20}$$

where x_{\perp} denotes spatial directions perpendicular to the surface normal \hat{n}_T . The second term, due to interactions, is given by

$$G_c^{(1)}(S; q, \omega) = \frac{\Omega}{2\pi} \frac{\Sigma^1(S; q, \omega)}{\hat{n}_S \cdot q} [\omega - \hat{n}_S \cdot q + i\eta \text{sgn}(\omega)]^{-2} \\ \approx \frac{\Omega}{2\pi} \frac{2\Lambda^{D-1}}{(2\pi)^D \chi^0(q, \omega)} [\omega - \hat{n}_S \cdot q + i\eta \text{sgn}(\omega)]^{-2} \tag{21}$$

where in the second line we have specialized to the particular case of finite x and singular interactions $V(q)$. The imaginary part of the fermion self-energy, which comes from $G_c^{(1)}$ in the limit of $|x| < 1$ for which quasiparticle damping occurs, may now be estimated by exponentiating (19) and expanding to first order in the imaginary part. The fermion propagator in real space is given by

$$iG_f(S; x, t) \equiv \langle \psi^\dagger(S; x, t)\psi(S; 0, 0) \rangle \\ = \frac{\Omega}{Va} e^{iks \cdot x} \exp\left(\frac{\pi}{\Omega^2} i[G_c(S; x, t) + G_s(S; x, t)]\right) \\ \approx \frac{i\Lambda^{D-1}}{(2\pi)^D} \frac{e^{iks \cdot x}}{x \cdot \hat{n}_S - t + i\delta \text{sgn}(t)} \left(1 + \frac{\pi i}{\Omega^2} \text{FT}[G_c^{(1)}]\right). \tag{22}$$

Here G_s is the free spin boson propagator. The correction to the fermion propagator in (k, ω) space is then given by

$$\delta G_f(k, \omega) = i \int_{-\infty}^{\infty} d\omega' \int_{-\lambda/2}^{\lambda/2} d^D q \frac{1}{\chi^0(q, \omega')} \\ \times \frac{1}{[\omega' - q_{\parallel} + i\eta \text{sgn}(\omega')]^2 [(\omega' - \omega) - (q_{\parallel} - k_{\parallel}) + i\eta \text{sgn}(\omega' - \omega)]} \tag{23}$$

where components of vectors parallel to the surface normal are denoted by, for example, $q_{\parallel} \equiv \mathbf{q} \cdot \hat{\mathbf{n}}_S$. The integral over q_{\parallel} may be performed by complex integration; no divergences occur since all the poles in the complex q_{\parallel} plane lie either on one side of the real axis or the other unless ω' lies between 0 and ω . Extracting the leading contribution to the fermion self energy at the pole we obtain (for $D = 2$)

$$\text{Im}\Sigma_f^{(1)}(\omega)|_{\text{pole}} \approx \frac{1}{N(0)(2\pi)^2} \text{sgn}(\omega) \left(\omega^2 \ln \frac{2\pi\omega^2\Lambda}{\lambda^2 k_F} - \frac{\omega^2}{2} \right) \quad (24)$$

which form we recognize immediately from previous work on the quasiparticle lifetime within the RPA approximation [9]. The quantity inside the braces is always negative since $2\pi\omega^2\Lambda \ll \lambda^2 k_F$. In three dimensions the calculation yields

$$\text{Im}\Sigma_f^{(1)}(\omega)|_{\text{pole}} \propto \frac{\lambda}{k_F^2} \omega^2 \text{sgn}(\omega) + O(\omega^3/k_F^2) \quad (25)$$

the appearance of the momentum cutoff λ in this expression coincides with that found in traditional Fermi liquid theory [10]. In the classic RPA calculation [11] the cutoff λ is replaced by an energy of order of the plasma frequency due to a different weighting of the high-energy states.

In the case of Coulomb interactions $V(\mathbf{q}) \propto |\mathbf{q}|^{1-D}$ the weight of the quasiparticle pole, Z_F , remains non-zero at the Fermi surface. In contrast, super-long-range interactions *can* destroy the quasiparticle pole. For example, Bares and Wen [12] studied a logarithmic interaction in two dimensions, given in momentum space by $V(\mathbf{q}) = g/q^2$, and showed that $Z_F = 0$ within RPA. Note that g is a momentum scale of order of the Fermi momentum and the plasmon gap is non-zero due to the super-long-range nature of the interaction and is given by $\omega_p \equiv \sqrt{gk_F/2\pi}$. In fact, this system may form a Wigner crystal; we ignore any such $2k_F$ instabilities in the following analysis. We confirm that the quasiparticle pole is destroyed in the super-long-range case via bosonization. One might be tempted to compute Z_F by using the Kramers–Kronig relation to derive the real part of the self-energy from the imaginary part estimated above. This procedure, however, is unreliable, as the Kramers–Kronig relations involve an integral over the entire energy range, whereas the above calculation ignores the emission of plasmons by quasiparticles. Instead we calculate the exact real-space fermion two-point function directly. To do this we compute the Fourier transform of (21) in the $|x| > 1$ region for which the self-energy (15) is purely real. For simplicity, we also expand the Lindhard function in powers of $1/x$:

$$\chi^0(x) = -\frac{N(0)}{2x^2} + i\eta + O(1/x^4). \quad (26)$$

If $\omega_p \gg \lambda$ the system does not see the plasmons and Fermi liquid behaviour is retained. In the opposite limit of $\omega_p \ll \lambda$, however, the plasmons destroy the Fermi liquid. The important point is that, in the case of the equal-time propagator, the integral over \mathbf{q} yields a logarithmic dependence on $x_{\parallel} \equiv \hat{\mathbf{n}}_S \cdot \mathbf{x}$ due to the factor $q^2 = q_{\perp}^2 + q_{\parallel}^2$ appearing in the denominator. Setting $\mathbf{r} = \mathbf{x}_{\perp} = 0$ we find

$$\begin{aligned} \ln G_f(S; x_{\parallel}) &\approx -\ln(x_{\parallel}) - ik_F x_{\parallel} + i \int_{-\lambda/2}^{\lambda/2} \frac{dq_{\parallel}}{2\pi} \int_{-\lambda/2}^{\lambda/2} \frac{dq_{\perp}}{2\pi} \\ &\times \frac{\exp(iq_{\parallel}x_{\parallel}) - 1}{q^2} \int_{|q|}^{\lambda} \frac{d\omega}{\pi} \frac{1}{1/g - k_F/(2\pi\omega^2)} \frac{1}{[\omega - q_{\parallel} + i\eta \text{sgn}(\omega)]^2}. \end{aligned} \quad (27)$$

Since $\omega > |q|$ in the integral, we make the approximation of replacing $[\omega - q_{\parallel} + i\eta \operatorname{sgn}(\omega)]^2$ with $\omega^2 + i\eta'$ in the denominator and then carry out the integrations. Upon exponentiating the boson Green function we obtain the Fermi quasiparticle propagator in configuration space:

$$G_f(S; x_{\parallel}) \approx \frac{(\lambda |x_{\parallel}|)^{-\zeta}}{x_{\parallel}} e^{-ik_{\parallel} x_{\parallel}} \quad |x_{\parallel}| \lambda \gg 1 \quad (28)$$

where

$$\zeta = \frac{1}{2} \sqrt{\frac{g}{2\pi k_F}}. \quad (29)$$

The appearance of the anomalous exponent ζ is a consequence of bosonization, which treats the interaction non-perturbatively and thus improves upon RPA which gives instead a logarithmic dependence on the system size [12]. Evidently, the quasiparticle pole has been destroyed by the super-long-range interaction. An evaluation of the fermion propagator in the opposite limit of $x = 0$ and at large times t , however, yields the usual t^{-1} Fermi liquid form as screening is effective at low frequencies. Likewise, thermodynamic properties that are defined in this equilibrium q -limit, such as the specific heat, also have the usual Fermi liquid form. Note that in either limit the propagator is odd under the combined CPT operation of $(x, t) \rightarrow -(x, t)$ and complex conjugation as demanded by Fermi statistics.

Repeating the calculation for a Coulomb interaction in either two or three dimensions we easily find that the propagator has the standard Fermi liquid form. The quasiparticle pole suffers no additional renormalization beyond the factor Z_k appearing in (2).

We examined the effects of long-range interactions on fermion liquids by bosonization in dimensions greater than one. We find that the Fermi liquid state is the only solution to the problem of a degenerate gas of fermions interacting via the Coulomb two-body interaction in two and three spatial dimensions. Bosonization allows us to go beyond an assumed Fermi liquid form for the quasiparticle propagator. Indeed, the fact that the bosonized Hamiltonian separates into charge and spin parts, $H = H_c + H_s$, leads to the possibility that, as in one dimension [4], the quasiparticle propagator might also exhibit spin-charge separation, especially in the case of the Coulomb interaction which is singular. Spin-charge separation in dimensions larger than one would, however, destroy the Fermi liquid as the key element, the existence of a pole in the single-particle Green function with non-zero spectral weight would be replaced by a branch cut and the pole would be destroyed. This does not happen because the location of the pole of the boson propagator is unchanged from its free value in the $\Lambda \rightarrow 0$ limit. Consequently, the spin and charge velocities are equal and both degrees of freedom propagate together in the usual quasiparticle form. On the other hand, the Fermi liquid form is destroyed in the case of the super-long-range interaction studied by Bares and Wen.

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